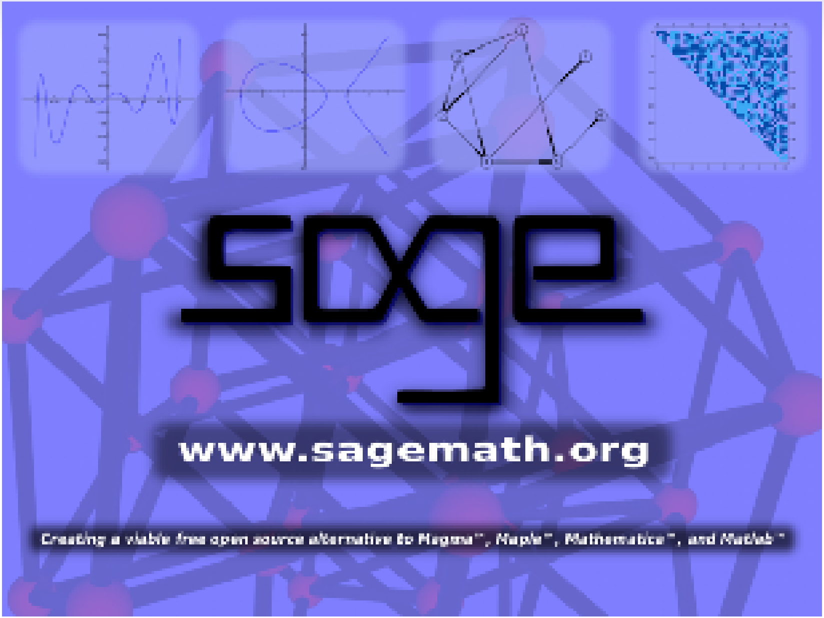
sage
www.sagemath.org
Creating a viable free open source alternative to Magma™, Maple™, Mathematica™, and Matlab™

Some very cool things about Sage

Or, why I am excited about Sage

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LaCIM Seminar
7 March 2008

Sage is a *distribution* of open-source software.

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Software included with Sage :

ATLAS	Automatically Tuned Linear Algebra Software
BLAS	Basic Fortan 77 linear algebra routines
Bzip2	High-quality data compressor
Cddlib	Double Description Method of Motzkin
Common Lisp	Multi-paradigm and general-purpose programming lang.
CVXOPT	Convex optimization, linear programming, least squares
Cython	C-Extensions for Python
F2c	Converts Fortran 77 to C code
Flint	Fast Library for Number Theory
FpLLL	Euclidian lattice reduction
FreeType	A Free, High-Quality, and Portable Font Engine

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Software included with Sage :

G95	Open source Fortran 95 compiler
GAP	Groups, Algorithms, Programming
GD	Dynamic graphics generation tool
Genus2reduction	Curve data computation
Gfan	Gröbner fans and tropical varieties
Givaro	C++ library for arithmetic and algebra
GMP	GNU Multiple Precision Arithmetic Library
GMP-ECM	Elliptic Curve Method for Integer Factorization
GNU TLS	Secure networking
GSL	Gnu Scientific Library
JsMath	JavaScript implementation of LaTeX

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Software included with Sage :

IML	Integer Matrix Library
IPython	Interactive Python shell
LAPACK	Fortan 77 linear algebra library
Lcalc	L-functions calculator
Libgcrypt	General purpose cryptographic library
Libgpg-error	Common error values for GnuPG components
Linbox	C++ linear algebra library
Matplotlib	Python plotting library
Maxima	computer algebra system
Mercurial	Revision control system
MoinMoin	Wiki

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Software included with Sage :

MPFI	Multiple Precision Floating-point Interval library
MPFR	C library for multiple-precision floating-point computations
ECLib	Cremona's Programs for Elliptic curves
NetworkX	Graph theory
NTL	Number theory C++ library
Numpy	Numerical linear algebra
OpenCDK	Open Crypto Development Kit
PALP	A Package for Analyzing Lattice Polytopes
PARI/GP	Number theory calculator
Pexpect	Pseudo-tty control for Python
PNG	Bitmap image support

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Software included with Sage :

PolyBoRi	Polynomials Over Boolean Rings
PyCrypto	Python Cryptography Toolkit
Python	Interpreted language
Qd	Quad-double/Double-double Computation Package
R	Statistical Computing
Readline	Line-editing
Rpy	Python interface to R
Scipy	Python library for scientific computation
Singular	fast commutative and noncommutative algebra
Scons	Software construction tool
SQLite	Relation database

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Software included with Sage :

Sympow	L-function calculator
Symmetrica	Representation theory
Sympy	Python library for symbolic computation
Tachyon	lightweight 3d ray tracer
Termcap	for writing portable text mode applications
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Plus additional optional packages

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Algebra	GAP, Maxima, Singular
Algebraic Geometry	Singular, Macaulay2
Arbitrary Precision Arithmetic	GMP, MPFR, MPFI, NTL, ...
Arithmetic Geometry	PARI, NTL, mwrank, ecm, ...
Calculus	Maxima, Sympy
Combinatorics	Symmetrica, MuPAD-Combinat*
Exact Linear Algebra	Linbox, IML
Graph Theory	NetworkX
Graphics	MatPlotLib, Tachyon3d
Group theory	GAP
Numerical Linear Algebra	GSL, Scipy, Numpy

Sage is a distribution of *free, open-source* software.

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— J. Neubüser (1993)

(started GAP in 1986)

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You have the freedom:

- to run the program, for any purpose.
- to study how the program works, and adapt it to your needs.
- to redistribute copies so you can help your neighbour.
- to improve the program, and release your improvements to the public, so that the whole community benefits.

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Also, you don't have to pay for it.

The Sage programming language is *Python*

Python is a powerful, modern, interpreted programming-language.

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- *Interpreted* means it works like Maple or Mathematica.

```
python: x = 17
python: x
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python: x**2
289
```

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```
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```

- It's *easy to learn*. Lots of free documentation.

```
http://diveintopython.org/
http://docs.python.org/tut/
```

The Sage programming language is *Python*

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- It's *easy to read and write*.

$$\left\{ 17x \;\middle|\; x \in \{0, 1, \ldots, 9\} \text{ and } x \text{ is odd} \right\}$$

```
python: [17*x for x in range(0,10) if x%2 == 1]
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- *Cython*: Python code $\longrightarrow$ compiled C code.

The Sage programming language is *Python*

"Google has made no secret of the fact they use Python a lot for a number of internal projects. Even knowing that, once I was an employee, I was amazed at how much Python code there actually is in the Google source code system."

— Guido van Rossum

(creator of Python)

Several ways to use Sage

- A library for Python scripts.

```
#!/usr/bin/env sage -python

import sys
from sage.all import *
```

Several ways to use Sage

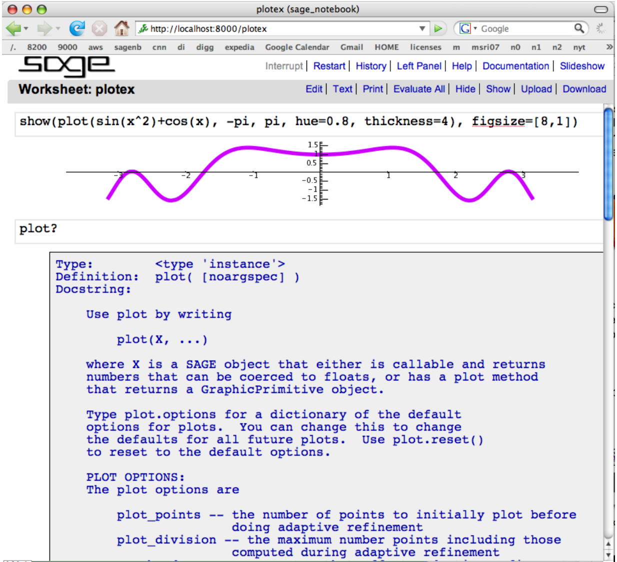
- Command line interface.

```
----------------------------------------------------------------
| SAGE Version 2.10.1, Release Date: 2008-02-02                 |
| Type notebook() for the GUI, and license() for information. |
----------------------------------------------------------------

sage: 17^2
289
sage: |
```

Several ways to use Sage

- Graphical notebook: online at `sagenb.org`



Sage plays well with LaTeX

LaTeX input:

```
\begin{sagesilent}
  var('s t')
  f = t^2*e^t-sin(t)
\end{sagesilent}

Let $f(t)=\sage{f}$. Then the Laplace tranform
of $f$ is: $\sage{f.laplace(t,s)}$.
```

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$\LaTeX$ output:

Let $f(t) = t^2e^t - \sin(t)$ *. Then the Laplace tranform of* f *is:* $\frac{2}{(s-1)^3} - \frac{1}{s^2+1}$.

Sage plays well with $\LaTeX$

$\LaTeX$ input:

```
Here is an example of a tree:
\sageplot{Graph({0:[1,2,3], 2:[5]}).plot()}
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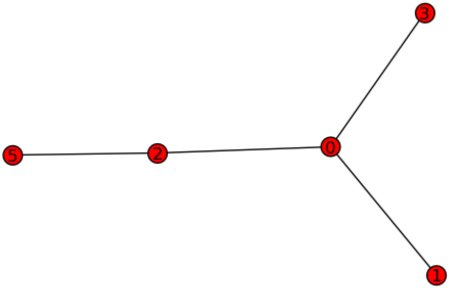

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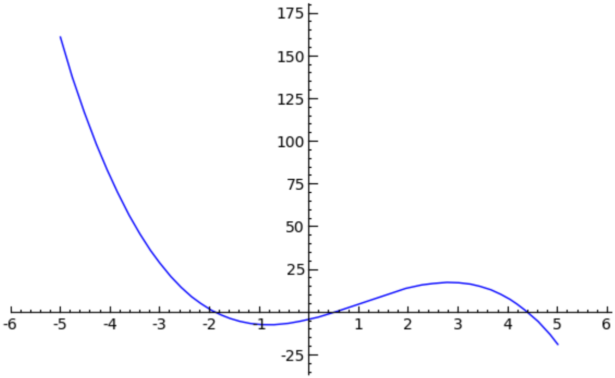


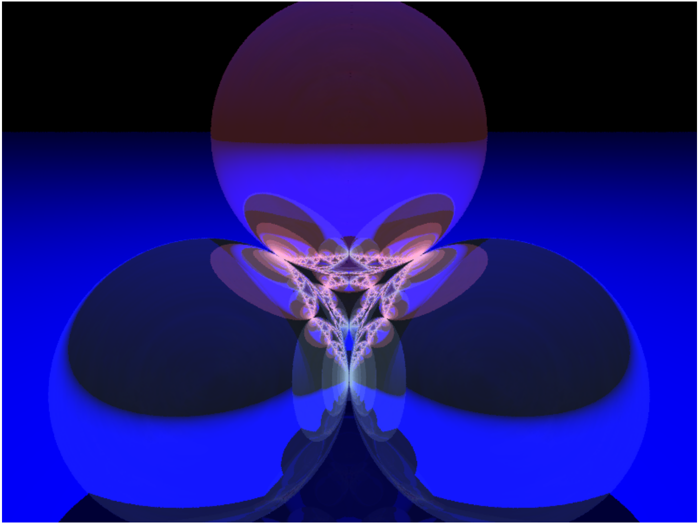
Sage plays well with LaTeX

LaTeX input:

```
\begin{sagesilent}
  t6 = Tachyon(camera_center=(0,-4,1), xres = 800, yres = 600, \
               raydepth = 12, aspectratio=.75, antialiasing = True)
  t6.light((0.02,0.012,0.001), 0.01, (1,0,0))
  t6.light((0,0,10), 0.01, (0,0,1))
  t6.texture('s', color = (.8,1,1), opacity = .9, specular = .95, \
             diffuse = .3, ambient = 0.05)
  t6.texture('p', color = (0,0,1), opacity = 1, specular = .2)
  t6.sphere((-1,-.57735,-0.7071),1,'s')
  t6.sphere((1,-.57735,-0.7071),1,'s')
  t6.sphere((0,1.15465,-0.7071),1,'s')
  t6.sphere((0,0,0.9259),1,'s')
  t6.plane((0,0,-1.9259),(0,0,1),'p')
\end{sagesilent}
\sageplot{t6}
```

Sage plays well with $\LaTeX$

$\LaTeX$ output:



The Sage community

- Many people have contributed to Sage (directly & indirectly).
- There are several mailing lists.

  `http://www.sagemath.org`

- IRC: `#sage-devel` on `freenode.org`.
- Developers are very friendly and helpful.

Let's use Sage

Let's use Sage

Demo 0: Get help.

- Start typing, then hit TAB.
- CommandName? for documentation and examples.
- CommandName?? for docs, examples and *source code*.

Let's use Sage

Demo 1: Interfaces.

```
sage: %maple

  --> Switching to Maple <--

maple: f := x -> x^2
f := proc (x) options operator, arrow; x^2 end proc
maple: D(f)(x)
2*x
maple: exit

  --> Exiting back to SAGE <--

sage:
```

Let's use Sage

Demo 1: Interfaces.

```
sage: %gap

  --> Switching to Gap <--

gap: s8 := Group( (1,2), (1,2,3,4,5,6,7,8) )
Group([ (1,2), (1,2,3,4,5,6,7,8) ])
gap: a8 := DerivedSubgroup( s8 )
Group([ (1,2,3), (2,3,4), (2,4)(3,5), (2,6,4), (2,4)(5
gap: Size( a8 ); IsAbelian( a8 ); IsPerfect( a8 )
20160
false
true
```

Let's use Sage

Demo 2: String manipulation

- Let $P_0 = \{\}$ and $P_n = \mathrm{PowerSet}(P_{n-1})$.

- Examples:
  - $P_1 = \{\{\}\}$
  - $P_2 = \{\{\{\}\}, \{\}\}$
  - $P_3 = \{\{\{\{\}\}, \{\}\}, \{\}, \{\{\}\}, \{\{\{\}\}\}\}$

- We want the words obtained from the elements in P_n by replacing each $\{$ with a and each $\}$ with b.

- Examples:
  - $P_1 \mapsto$ [ab] .
  - $P_2 \mapsto$ [ab, aabb] .
  - $P_3 \mapsto$ [ab, aabb, aaabbb, aaabbabb] .

Let's use Sage

```
# Import a module (library)
import string

# Define a function to generate the sets
def P(n):
    if n == 0:
        return Set([])
    else:
        return Subsets(P(n-1))

# Define a function to the replacing.
f = lambda x : str(x).translate(string.maketrans('{}','ab'),', ')

# Do a list comprehension to combine them.
words = lambda n : [f(x) for x in P(n)]
```

Let's use Sage

Demo 3: Sloane

```
sage: seqs = sloane_find([1,1,2,3,5,8,13],1)
sage: for x in seqs:
....:     print x[1]
Fibonacci numbers: F(n) = F(n-1) + F(n-2), F(0) = 0,
F(1) = 1, F(2) = 1, ...
```

Let's use Sage

Demo 4: Play with Partitions

- Set of Partitons
  - `P = Partitions(6)`
  - `P.list()`
  - `P.count()`
  - `P.<tab>`

- individual partitons
  - `nu = Partitions(6).random(); nu`
  - `nu = Partition([3,2,2,1])`
  - `print nu.ferrers_diagram()`
  - `nu.hook_lengths()`
  - `nu.conjugate()`
  - `nu.hook_product(x)`
  - `nu.hook_product(1)`
  - `nu.<tab>`

Let's use Sage

Demo 5: Play with Symmetric Functions

- Help: `SymmetricFunctionAlgebra?`
- Power basis:
  `p = SymmetricFunctionAlgebra(QQ, basis='power'); p`
- Expand: `p([3]).expand(4)`
- Elementary basis: `e = SFAElementary(QQ)`
- Monomial basis: `m = SFAMonomial(QQ)`
- Homogeneous basis: `h = SFAHomogeneous(QQ)`
- Schur basis: `s = SFASchur(QQ)`
- Dual basis: `m.dual_basis() is h`
- Omega: `m([2,2,1]).omega()`
- Change of basis: `m(h([3]))`
- Change of basis matrix: `h.transition_matrix(m,4)`
- Plethysm: `s([3])(s([3,2]))`

Let's use Sage^{++}

Demo 6: Play with Jack and Macdonald Polynomials

```
sage: H = MacdonaldPolynomialsH(QQ); H
sage: s = SFASchur(H.base_ring()); s
sage: s(H([2]))
sage: _.expand(3)
sage: J = JackPolynomialsJ(QQ,t=1); J
sage: s = SFASchur(J.base_ring()); s
sage: nu = Partitions(7).random(); nu
sage: J(nu)
sage: s(J([3,2,2,1]))
sage: nu.hook_product(1)
```

Let's use Sage^{++}

Demo 7: Manipulate

```
sage: @manipulate
sage: def _(a=(0,1)):
....:     x,y = var('x,y')
....:         show(plot3d(sin(x*cos(y*a)), \
....:              (x,0,5), (y,0,5)), figsize=4)
```

Let's use Sage^{++}

Demo 8: Posets

```
sage: P = Poset([[1,2],[4],[3],[4],[]]); P
sage: P.antichains()
sage: P.show()
sage: P.is_meet_semilattice()
sage: P.is_graded()
sage: Pi = PosetOfIntegerPartitions(5); Pi
sage: Pi.show()
sage: B = BooleanLattice(5); B
sage: B.show()
sage: PosetOfRestrictedIntegerPartitions(7).show()
```

Let's use Sage

Demo 9: Rubik's cube

```
sage: C = RubiksCube().scramble()
sage: C.show()
sage: C.show3d()
sage: C.solve()
```

Let's use Sage

Demo 10: Linear Algebra & Sudoku Solver

```
sage: A = matrix(ZZ,9,[5,0,0, 0,8,0, 0,4,9, \
                       0,0,0, 5,0,0, 0,3,0, \
                       0,6,7, 3,0,0, 0,0,1, \
                       1,5,0, 0,0,0, 0,0,0, \
                       0,0,0, 2,0,8, 0,0,0, \
                       0,0,0, 0,0,0, 0,1,8, \
                       7,0,0, 0,0,4, 1,5,0, \
                       0,3,0, 0,0,2, 0,0,0, \
                       4,9,0, 0,5,0, 0,0,3]); A
sage: A.determinant()
sage: A.minpoly()
sage: sudoku(A)
```