

04 - Symmetric Functions

Symmetric Functions

Power sums basis

```
p = SymmetricFunctionAlgebra(QQ, basis='power')
```

```
p
```

Symmetric Function Algebra over Rational Field, Power symmetric functions as basis

```
a = p([3]); a
```

p_3

```
a.expand(4)
```

$$x_0^3 + x_1^3 + x_2^3 + x_3^3$$

```
a.expand(4, alphabet='x,y,z,t')
```

$$x^3 + y^3 + z^3 + t^3$$

Homogeneous functions basis

```
h = SFAHomogeneous(QQ); h
```

Symmetric Function Algebra over Rational Field, Homogeneous symmetric functions as basis

```
h([3]).expand(2, alphabet='x,y')
```

$$x^3 + x^2y + xy^2 + y^3$$

```
h(a)
```

$$h_{1,1,1} + (-3)h_{2,1} + 3h_3$$

Schur functions basis

```
s = SFASchur(QQ); s
```

Symmetric Function Algebra over Rational Field, Schur symmetric functions as basis

```
s([3,2]).expand(3)
```

$$x_0^3x_1^2 + x_0^2x_1^3 + x_0^3x_1x_2 + 2x_0^2x_1^2x_2 + x_0x_1^3x_2 + x_0^3x_2^2 + 2x_0^2x_1x_2^2 + 2x_0x_1^2x_2^2 + x_1^3x_2^2 + x_0^2x_2^3 + x_0x_1x_2^3 + x_1^2x_2^3$$

```
s(a)
```

$$s_{1,1,1} + (-1)s_{2,1} + s_3$$

Plethysm

```
p([3,2])(s([2,1]))
```

$$\frac{1}{9}p_{3,3,3,2,2,2} + \left(-\frac{1}{9}\right)p_{6,3,3,3} + \left(-\frac{1}{9}\right)p_{9,2,2,2} + \frac{1}{9}p_{9,6}$$

Other Symmetric Functions