Worksheet 4 - Calculus, Plotting and Interact

Calculus, plotting & interact

Some differentiating and plotting

Exercise: Let $f(x) = x^4 + x^3 - 13x^2 - x + 12$. Define f as a symbolic function.

Exercise: Plot f on the domain $-4.5 \leq x \leq 3.5$.

Exercise: Find numerical approximations for the *critical values* of f by taking the derivative of f and using the **find_root** method. (*Hint:* plot the derivative.)

Exercise: Find numerical approximations for the *critical values* of f by taking the derivative of f and using the **roots(ring=RR)** method. (Here, **RR** stands for the real numbers.) Are there any roots over the ring of rationals (**QQ**)?

Exercise: Compute the equation $y = mx + b$ of the tangent line to the function f at the points $x = -1$ and $x = 2$.

Exercise: Write a function that takes x as an argument and returns the equation of the tangent line to f through the point x.

Exercise: Write a function that takes x as an argument and plots f together with the the tangent line to f through the point x. Make the line *red*.

Exercise: Convert the function you created above into an **@interact** object. Turn the argument x into a **slider**. (*Hint:* see the documentation for **interact** for examples on creating **sliders**.)

Differential Equations

Using *symbolic functions* and the command **desolve** in Sage, we can define and solve differential equations. Here is an example.

We will solve the following differential equation:

$$y'(t) + y(t) = 1.$$

First we define the *variable* t:

```
var('t')
```

```
t
```

Next, we define the *symbolic function* y:

```
y = function('y', t)
y
```

```
y(t)
```

We can now create the differential equation:

```
diff_eqn = diff(y,t) + y - 1
diff_eqn
```

```
diff(y(t), t, 1) + y(t) - 1
```

We can use the **show** command to typeset the above equation to make it easier to read:

```
show(diff_eqn)
```

$$\frac{\partial}{\partial t} y\left(t\right) + y\left(t\right) - 1$$

Finally, we use the **desolve** command to solve the differential equation:

```
soln = desolve(diff_eqn, y)
soln
```

```
e^(-t)*(e^t + c)
```

```
show(soln)
```

$e^{-t}\left(e^{t}+c\right)$

Exercise: Find and plot the solution to the differential equation

$$y'(t) = y(t)^2 - 1$$

with the intial condition $y(0) = -2$. (*Hint:* see the documentation of the **desolve** command for dealing with initial conditions.)

Exercise: Find and plot the solution to the differential equation

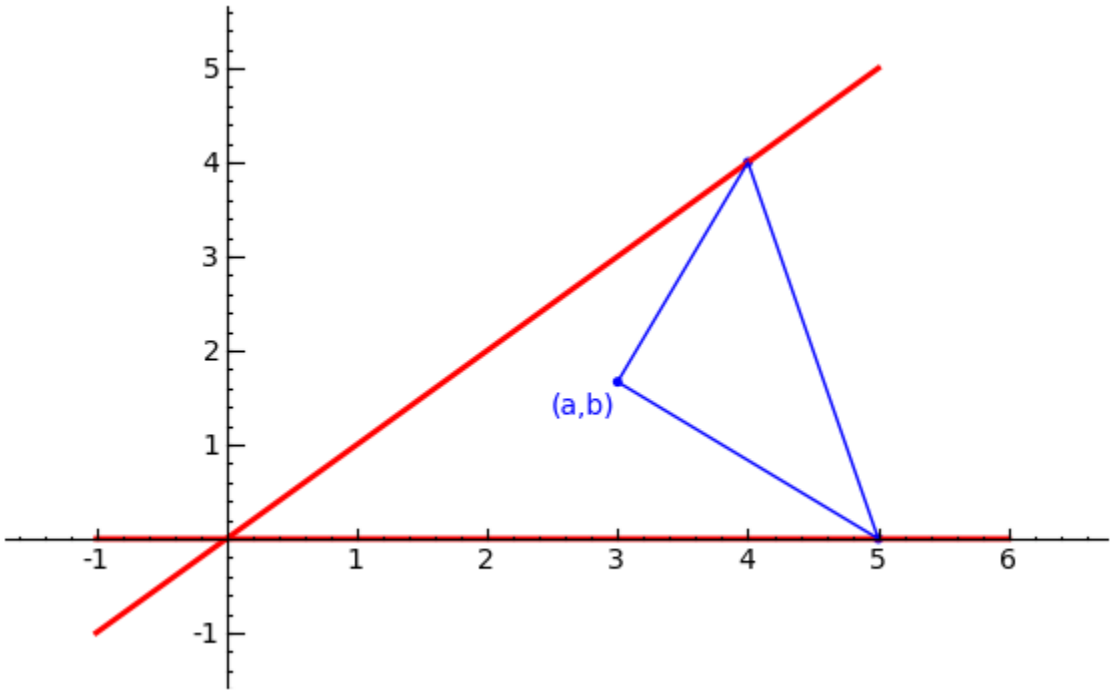
$$ty'(t) + 2y(t) = \frac{e^t}{t}$$

with initial conditions $y(1) = -2$. (*Hint:* see the documentation of the **desolve** command for dealing with initial conditions.)

[David Joyner, *Introductory Differential Equations using SAGE*]

Problem

Let $a > b > 0$ be fixed real numbers and form a triangle with one vertex on the line $y = x$, one vertex on the line $y = 0$ and the third vertex equal to (a, b).



Find the coordinates of the vertices that minimize the perimeter of the triangle (remember that (a,b) is fixed!). What is the perimeter?